Bayesian Probability Theory and Markov Chain Monte Carlo

G. Larry Bretthorst
Biomedical Magnetic Resonance Laboratory,
Department of Radiology,
Washington University, St. Louis, MO 63110

Abstract

Bayesian probability theory can be traced back nearly 250 years, yet as recently as 40 years ago, most applications of Bayesian probability theory were restricted to problems for which analytic solutions were available. The advent of computers changed that and allowed more complex problems to be addressed. Problems involving only a few parameters could be solved using numerical integration to evaluate the marginal probabilities; more complex problems were approximated using searching algorithms and the Laplace approximation to evaluate the integrals. While this was a great advance, these techniques failed when the joint posterior probability was not sharply peaked, was multimodal, or involved more than about 100 parameters. Thus, there were many problems that could not be adequately solved using probability theory. Recently, the recognition that Markov chain Monte Carlo techniques can be used to sample the Bayesian posterior probability has made it possible to apply probability theory to obtain the solution to almost any problem. Markov chain Monte Carlo is not a single method; rather it is broad class of techniques that allow one to generate random numbers sampled from a target distribution. In this talk, we will concentrate on describing one such technique, Metropolis-Hastings with simulated annealing, that is relatively simple and capable of sampling any Bayesian posterior probability, including those associated with model selection calculations. We will describe the algorithm and illustrate the steps necessary to implement, control, monitor and parallelize it using examples taken from calculations performed at the Biomedical Magnetic Resonance Laboratory.