Newton, Einstein, Jeffreys and Bayesian Model Selection

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Abstract

In [1] Jefferys and Berger apply Bayesian model selection to an intriguing problem: That of choosing between rival theories, in particular between Einstein's theory of general relativity (GR) and Newtonian gravity (NG). [1] presents a 1921 debate that occured between Harold Jeffreys and Charles Poor regarding the observed 43 arc second per century anomalous perhelion precession of Mercury. GR made a precise prediction of 42.98 arc seconds per century while proponents of NG suggested several physical mechanisms that were eventually refuted except for the possibility that the inverse square law of distance could be modified to be proportional to $\frac{1}{r^{2+\epsilon}}$.

Using Bayes Factors (BF) and experimental data available in 1921, [1] shows that GR is preferable to NG by a factor of about 25 to 1. A scale for BF used by Harold Jeffreys [2], suggests that this is positive to strong evidence for Einstein over Newton but it is not very strong or even overwhelming. At this point Jefferys and Berger leave us hanging by not taking the analysis to its logical conclusion.

In this work we calculate the BF from the period 1921 till 1993. By 1960 we see that the BF, due to better data gathering techniques and advances in technology, had reached a factor of greater than 100 to 1, making GR strongly preferable to NG and by 1990 the BF reached 1000:1. Ironically while BF had reached a state of near certainty even in 1960 rival theories of gravitation were on the rise - notably the Brans–Dicke (BD) scalar–tensor theory of gravity [3]. The BD theory is postulated in such a way so that depending on the the value of a scalar parameter ω, the BF would either mildly favor either GR or BD (i.e., BF ≈ 1) or it would approach unity with certainty, at which point either theory would be prefered, i.e., it is a theory that cannot lose. Yet scientists prefer GR over BD - does this mean Bayesian model selection needs to be overthrown? As it turns out this paradox can be resolved by utilizing cogent prior information from the physics.

References:

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Key Words: Bayes Theorem, Model Selection, General Relativity, Newtonian Gravity, Harold Jeffreys, Brans–Dicke theory