SOFTENING A HARD BOUND TO A PRIOR PDF — A PROBLEM IN GEOMAGNETISM

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Abstract

The Earth's magnetic field is generated within the fluid core of the Earth, and measurements of the vector field can be taken above the Earth's surface. In order to gain intuition about the generation process, and to compare different theories, it is of interest to reconstruct the morphology of the field at the interface between the core and the overlying solid mantle, 2900km beneath the Earth's surface. This problem of inference has attracted a great deal of attention over the last 2 decades, because of the existence of prior information about the underlying Earth model $\mathbf{m}$, taking the form of a quadratic bound on $\mathbf{m}$

$$Q(\mathbf{m},\mathbf{m}) = \sum_{i}^{\infty} m_i^2 < q \tag{1}$$

where m_i are parameters associated with an orthonormal basis for $\mathbf{m}$ and q is known. This prior information is sufficient to enable estimates of linear functionals of the model to be made, along with realistic error bounds.

Considerable debate has centred around the softening of a hard bound, such as (1), into a prior probability density. The problem lies in the fact that the true Earth model is strictly infinite dimensional; Backus (1989) showed that the softening of a hard bound into a prior pdf injects much more information than is contained in the bound, and some information that is contradictory to the bound. As a result, it is claimed that Bayesian calculations based on the softening of hard quadratic bounds are suspect. Consequently, geomagnetists have become paralysed by this conclusion, and no Bayesian calculations at all have been performed, when perhaps much could have been learned.

Here we present a pragmatic approach to the softening of a bound such as (1), which appears to us to be a suitable approximation to (1) in a finite dimensional subspace. This could be the basis for Bayesian calculations in the geomagnetic inference problem, to which other supposedly safer calculations could be compared.

References:

[1] Backus, G. E., Geophys. J. Int. **94**, 249 (1988).