STOCHASTIC SPECTRAL METHODS FOR EFFICIENT BAYESIAN SOLUTION OF INVERSE PROBLEMS

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Abstract

The Bayesian setting for inverse problems provides a rigorous foundation for inference from noisy data and uncertain forward models, a natural mechanism for incorporating disparate prior sources of information, and a quantitative assessment of uncertainty in the inferred results. Obtaining useful information from the posterior density—e.g., computing expectations via Markov Chain Monte Carlo (MCMC)—may be a computationally expensive undertaking, however. For complex and high-dimensional forward models, such as those that arise in inverting systems of PDEs, the cost of likelihood evaluations may render MCMC sampling prohibitive.

We explore the use of polynomial chaos (PC) expansions [1,2] for spectral representation of stochastic model parameters in the Bayesian context. The PC construction employs orthogonal polynomials in i.i.d. random variables as a basis for the space of square-integrable random variables. We use a Galerkin projection of the forward operator onto this basis to obtain a PC expansion for the outputs of the forward problem [3]. Evaluation of integrals over the parameter space is recast as MCMC sampling of the random variables underlying the PC expansion, which may lead to significant cost savings in the evaluation of the likelihoods.

We evaluate the utility of this technique on a transient diffusion problem arising in contaminant source inversion; specifically, we estimate source parameters—the location, size, strength, and time interval of a contaminant release—from sparse pointwise measurements of the field. The accuracy and efficiency of MCMC sampling is examined with respect to key aspects of the formulation: the model parameter distribution employed in the initial spectral reformulation of the forward problem; the order of the PC representation; and the choice of the PC basis. We contrast the computational cost of the new scheme with that of direct MCMC sampling.

References:

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