

# Sequential Marginal Likelihood Change Detection: Sequential Monte Carlo Approach

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## Abstract

Consider sequential data

$$y_{1:t} := (y_1, \dots, y_t) = (y_{1:t-1}, y_t), \quad t = 1, 2, \dots, \quad y_{1:1} := y_1$$

from an unknown system with changing parameter(s). A plausible likelihood function can be of the form  $P(y_t | \theta_t, \beta_t, \mathcal{H})$  defined through “basis” functions parameterized by  $\theta_t$ , with  $\beta_t$  being a possible hyperparameter, and  $\mathcal{H}$  represents the underlying model structure. We will consider the stochastic online learning dynamics for parameters and hyperparameters:

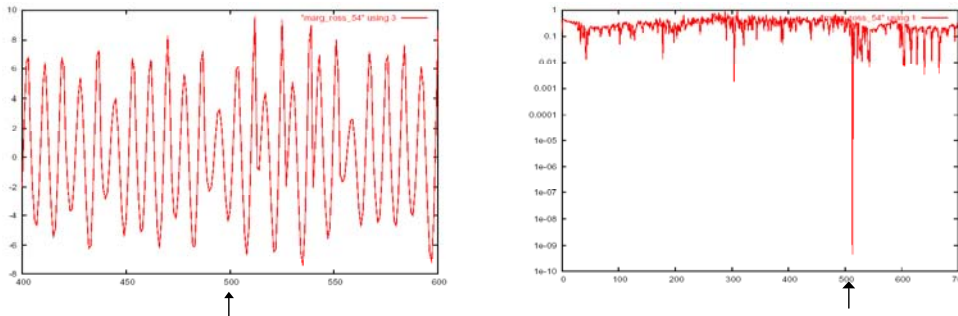
$$P(\theta_t, \gamma_t, \beta_t | \theta_{t-1}, \gamma_{t-1}, \beta_{t-1}, y_{t-1}, \mathcal{H}) = P(\theta_t | \theta_{t-1}, \gamma_t, y_{t-1}, \mathcal{H}) P(\gamma_t | \gamma_{t-1}, \mathcal{H}) P(\beta_t | \beta_{t-1}, \mathcal{H})$$

where  $\gamma_t$  is another hyperparameter that controls the  $\theta_t$ -dynamics.

This study attempts to perform online change detection by examining the time dependency of the *sequential marginal likelihood*:

$$t \mapsto \begin{cases} P(y_t | y_{1:t-1}, \mathcal{H}) = \iint P(y_t | \theta_t, \beta_t, \mathcal{H}) P(\theta_t, \alpha_t | y_{1:t-1}, \mathcal{H}) d\theta_t d\alpha_t \\ P(y_{1:t} | \mathcal{H}) = \prod_{s=1}^t P(y_s | y_{1:s-1}, \mathcal{H}) \end{cases} \quad (1)$$

with  $\mathcal{H}$  fixed, where  $\alpha_s := (\beta_s, \gamma_s)$  and utilizing Sequential Monte Carlo for evaluation.



Example: Only available observation in this example is a one dimensional time series data (left) from a higher order nonlinear dynamical system (the Rossler system) where an internal change occurs at  $t = 500$  indicated by the arrow. Observe that the change in the observed data is rather subtle. The figure on the right is  $\log P(y_t | y_{1:t-1}, \mathcal{H})$  which appears to indicate possibility of online change detection of unknown system via (1).

References:

- [1] A. Doucet et. al, eds., *Sequential Monte Carlo in Practice*, Springer, (2001)
- [2] D. J. C. MacKay, *Information Theory, Inference, and Learning Algorithms*, Cambridge University Press, (2004)
- [3] T. Matsumoto, et. al, *IEEE Transactions on Signal Processing*, 49, 2138 (2001)