ON CLASSICAL, BAYESIAN AND FUZZY HYPOTHESES TESTING

A. Mohammadpour[1,2], and A. Mohammad-Djafari[1]
(1) L2S, CNRS Supélec Univ. Paris-sud, Gif-sur-Yvette, France
(2) Amirkabir University of Technology, Tehran Polytechnic, Iran

Let $X_1, \cdots, X_n$ be independent and identically distributed with density function $f(x|\theta)$, where θ is a one dimensional parameter. Consider testing simple versus simple hypotheses

$$\begin{cases} H_0 : \theta = \theta_0 \\ H_1 : \theta = \theta_1 \end{cases},$$

where θ_0 and θ_1 are fixed numbers, based on a random sample. One can find the best solution for this problem in the different frameworks as follows:

Test	θ	Critical region (by)	Ref. [3]
Classic	fixed and unknown parameter	Neyman-Pearson	p.243
Bayes	random variable with known prior	Likelihood Ratio	p.227
Empirical Bayes	random variable with unknown prior	Likelihood Ratio	p.483

Now consider fuzzy hypotheses

$$\begin{cases} H_0 : \theta \simeq \theta_0 \ \ (\theta \text{ is approximatly } \theta_0) \\ H_1 : \theta \simeq \theta_1 \ \ (\theta \text{ is approximatly } \theta_1) \end{cases},$$

where $\theta \simeq \theta_i,\ i = 0, 1$ are expressed by two membership functions $m_0(\theta)$, $m_1(\theta)$ in fuzzy community and by two prior probability laws $\pi_0(\theta)$ and $\pi_1(\theta)$ in Bayesian community. A few authors had tried to find the best test for testing fuzzy hypotheses, [1,2,4]. In this paper we show that the best test for fuzzy hypotheses in the Bayesian framework is simply equivalent to Neyman-Pearson Lemma in the classical statistics.

Key Words: Calssic, Bayes and empirical Bayes test, fuzzy hypotheses, Neyman-Pearson lemma, likelihood ratio test.

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