INTRODUCTION TO THE INFORMATION IN METRIC SPACE

M. Pappalardo[1]
(1) DIMEC Department of Mechanical Engineering,
University of Salerno
Via P. don Melillo, 84084 Fisciano (SA). Italy
(mpappalardo@unisa.it)

Abstract

The idea of information, in the classic theories of Fisher and Wiener-Shannon, is a measure only of probabilistic and repetitiveness events. The idea of information is broader than the probability. It is possible the introduction of a Theory of Information for events not connected to the probability therefore for non repetitive events. In this paper the Wiener–Shannon's axioms are extended to the non-probabilistic and repetitiveness events.

On the basis of so called Laplace's Principle of insufficient knowledge, the *MaxInf* principle is defined for choose solutions in absence of knowledge. The value of information, as a measure of equality of data among a set of values, is applied in numeric analysis as method for approximation of data. The extension of information in metric space is obtained defining the next axioms in the following way:

Let Ω to be the field of all events ω, probabilistic or non-probabilistic, and $\Im$ a class of parts of Ω, $\Im \subset \wp_{arts}(\Omega)$. With $A \subset \Im$ we can assume the next two axioms:

AXIOM I: The value of information *J(A)* is always non negative: $J(A): \Im \to \mathrm{R}^+$

AXIOM II: The value of information *J(A)* is monotonous in regard to inclusion: $\forall A, B \in \Im \quad , \quad B \subset A \quad , \quad J(B) \geq J(A)$

Now the construction of new algorithms it is possible in terms of information, founded only on the first and second axioms. For independent events it is opportune to assume a third axiom:

AXIOM III: If the events $A, B \in \Im$ are independent for all the values of information we have: $\forall A, B \in \Im \quad \mathrm{J}(B \cap A) = J(B) + J(A)$.

The third axiom shows that when we are in presence of independent events it is possible to add up information. If Ω is a certain event and ϕ the impossible event than, for an universal validity of $J(A)$ and $J(\phi)$, for all $\Omega, \Im$ and J must be: $J(\Omega) = 0 \ , \ J(\phi) = +\infty$

The expression $J(\Omega) = 0$ means that Ω is a certain event without needs of information. The knowledge of ω is not given by its coordinates in Ω, but it is possible only to assert that ω is limited in a subset $A_i \in \Im$. If $d(A_i)$ is the diameter of set A_i, than, more is the precision of measures, less is the measure of diameter of event. Using the axioms I, II, and III it is possible to develop models for information very useful in applications. For every event $A \in \Im$ we can have a measure of information using the mathematical expression :

$$J(A) \overset{def}{=} \Psi\left[\sum |d(A_i)|\right] \qquad ; \qquad J(A_i) \propto \frac{1}{|d(A_i)|}$$

One application of the *MaxInf* principle is in the problems of approximation as criteria to find polynomials to represent a given set $E = \{(x_i, y_i),..\}$ of empirical points. It is possible to have solutions with approximation on the basis of maximum information *(MaxEnt or MaxInf)*.

Keywords: MaxEnt, Probability, Entropy, Classification, Approximation