VARIATIONAL BAYESIAN LEARNING FOR WAVELET INDEPENDENT COMPONENT ANALYSIS

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Abstract

In an exploratory approach to data analysis, it is often useful to consider the observations as generated from a set of latent generators or "sources" via a generally unknown mapping. For the noisy overcomplete case, where we have more sources than observations, the problem becomes extremely ill-posed. Solutions to such inverse problems can, in many cases, be achieved by incorporating prior knowledge about the problem, captured in the form of constraints [1].

This setting is a natural candidate for the application of the Bayesian methodology, allowing us to incorporate "soft" constraints in a natural manner. The work described in this paper is mainly driven by problems in functional magnetic resonance imaging of the brain, for the neuro-scientific goal of extracting relevant "maps" from the data. This can be stated as a 'blind' source separation problem. Recent experiments in the field of neuroscience [2] show that these maps are *sparse*, in some appropriate sense. The separation problem can be solved by independent component analysis (ICA), viewed as a technique for seeking sparse components, assuming appropriate distributions for the sources. We derive a hybrid wavelet-ICA model, transforming the signals into a domain where the modeling assumption of sparsity of *the coefficients* with respect to a dictionary is natural [3]. We follow a graphical modeling formalism, viewing ICA as a probabilistic *generative* model. We use hierarchical source and mixing models and apply Bayesian inference to the problem. This allows us to perform model selection in order to infer the complexity of the representation, as well as automatic denoising. Since exact inference and learning in such a model is intractable, we follow a variational Bayesian mean-field approach in the conjugate-exponential family of distributions, for efficient unsupervised learning in multi-dimensional settings. The performance of the proposed algorithm is demonstrated on a variety of experiments.

References:

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