OPTIMAL DISCRETIZATION RESOLUTION IN ALGEBRAIC IMAGE RECONSTRUCTION

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Abstract

In algebraic image reconstruction, the unknown image is represented by a weighted sum of a set of basis functions. We refer to the number of basis functions used in the series expansion as the discretization resolution. In this work, the problem of finding the optimal resolution is treated as a model order selection problem and two different approaches are investigated. We focus on tomographic imaging problems where the underlying linear inverse problem is ill-posed and some form of regularization is needed. A general form of Tikhonov regularization is taken as the choice of regularization method. Even in the regularized case the reconstruction resolution has strong implications on the quality of reconstruction. If the selected resolution is too low, the discretization grid is unable to represent the underlying image. And if the selected resolution is too high, the representation will fit noisy features that are either physically impossible or subject to misinterpretation.

The first approach is using Mallow's C_L method or generalized cross validation which have been previously used as regularization parameter selection methods (see Ch. 7 of [1]). For each of the two methods, we propose a joint estimator of regularization parameter and discretization resolution. By generalizing the results of [2] for model selection in the regularized case, we provide and interpret conditions for asymptotic optimality of the resolution estimators. The second approach is a Bayesian estimator of the model order first introduced in [3] where the signal and noise covariances were assumed to be diagonal with unknown variance. Here we assume a general covariance matrix for both signal and noise which are known beforehand. Furthermore, we use a complexity-penalizing prior and the concept of effective number of parameters from the neural computation literature.

Numerical experiments focus on a space imaging application whereby the goal is to reconstruct a feature in the ionosphere from a set of limited-angle tomographic observations measured either by GPS occultations or ground-based receivers.

References:

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